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Mach Wave Radiation from High-Speed Jets

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Extensive experimental evidence is now available to support the observation that there are two components of jet mixing noise. They are the fine-scale turbulence noise and the noise from the large turbulence structures of the jet flow. The large turbulence structure's noise radiates primarily in directions with a large inlet angle around the downstream axis of the jet. The fine-scale turbulence noise dominates in the sideline and upstream directions. This study investigates the mechanism by which large turbulence structures radiate noise. It is believed that the mechanism is Mach wave radiation. Theoretical model results and physical reasoning are presented to support the Mach wave mechanism. They are further supported by experimental measurements both in the far field and in the near acoustic field. These measurements include peak noise direction, noise-source distribution along the jet column, and near-field pressure-contour pattern. A signature pattern of the near-field pressure contours associated with Mach wave radiation is identified.

I. Introduction

THERE is now a large body of experimental evidence to support the proposition that there are two components of turbulent mixing noise from high-speed jets. They are the noise from the large and the fine-scale turbulence of the jet flow. Figure 1 shows the noise sources and their sound fields as proposed in [1–3]. The large turbulence structure's noise radiates to directions with a large inlet angle around the downstream jet axis. The fine-scale turbulence noise is dominant in the sideline and upstream directions. In the core region of a jet and extending to another half-to-one core length, optical observations indicate that there is a distinct separation of turbulence scales. The large turbulence structures of a jet are produced by Kelvin–Helmholtz instabilities of the jet mean flow. They are found to dominate the dynamics of the mixing layer of high-speed jets. The cascade process, which transforms low-wave-number (large-scale) turbulence motion to high wave numbers (small scale) and would have produced a monotonically smooth spectrum, does not have enough time to act in the first two core lengths of the jet. The result is the existence of two distinct scales of turbulence in the most energetic portion of the jet, creating two components of noise with vastly different characteristics. That this is the case is supported by optical observations. Figure 2, taken by Tam et al. [4], shows a spark schlieren picture of a Mach 1.3 jet. The presence of blobs of fine-scale turbulence in the potential core of the jet is clearly shown. Figure 3 is a pulsed-laser picture of a Mach 1.4 jet taken by Thurow et al. [5]. The presence of large turbulence structures in the potential core region of the jet is evident. The two types of turbulence structures with distinctly different scales, indeed, coexist in the potential core region of high-speed jets.

The first evidence in support of the proposition of two components of jet mixing noise is the recognition by Tam et al. [3] that it is possible to fit jet noise spectra to two seemingly universal similarity spectra. Their peaky spectrum fits all jet noise spectra at large inlet angles regardless of jet Mach number and jet temperature ratio. Their second spectrum is very broad and it fits all measured jet noise spectra in the forward and sideline directions, again regardless of jet Mach number and temperature ratio. That this is true has been verified by the works of Viswanathan [6], Dahl and Papamoschou [7], and Tam [8]. Figure 4 shows the two similarity spectra of Tam et al. [3]. Figure 5 is a demonstration of how well these two spectra fit the measured data of three jets with Mach numbers of 0.5, 1.0, and 2.0. The temperature ratio of these jets is $T_r/T_a = 3.2$ (reservoir temperature to ambient temperature). The unprocessed data are from Seiner et al. [9] and Viswanathan [10]. The two similarity spectra are, by and large, empirical. It is therefore useful to establish their range of applicability. The present comparison with data suggests that these spectra do fit jet noise spectra well, at least up to a temperature ratio of 3.2. This upper limit may, however, be moved up when even-higher-temperature jet noise data become available in the future.

Directivity data in the work of Tam et al. [4] show two distinct sectors. In the sector containing the upstream and sideline directions, the directivity increases steadily with inlet angle. In the downstream sector, in which the noise level is most intense, the directivity increases rapidly with inlet angle. It peaks and then drops off. The two distinct directivity patterns indicate two very different noise sources. Data on the directivity of the Strouhal number fD/U_j at the peak of the jet noise spectra (peak Strouhal number) further reinforce the belief of two noise components. In the upstream and sideline sectors, the measured directivity found by Tam et al. shows a continuous but slow increase in peak Strouhal number with inlet angle. However, the peak Strouhal number decreases with increase in temperature ratio. In the downstream sector, the behavior is opposite. The peak Strouhal number decreases with increase in inlet angle and is almost completely unaffected by change in temperature ratio. This again suggests that there are two distinct sound fields radiated from high-speed jets.

It is well known that the overall sound pressure level (OASPL, denoted by I) of jet noise has a power-law dependence on jet velocity [11]. In dimensionless form, the power law may be written as

$$\frac{I}{p_\infty^2} = A\left(\frac{U_j}{a_\infty}\right)^n \frac{1}{(\frac{r}{D})^2}$$

where U_j is the jet exit velocity; p_∞ and a_∞ are the ambient pressure and sound speed, respectively; r is the radial distance to the jet; D is the jet diameter; n is the power-law exponent; and A is the proportionality factor. By examining a large bank of jet noise data measured by Seiner et al. [9] and those by Viswanathan [10], Tam et al. [4] were able to present data on the dependence of the generalized power exponent $n(\theta, T_r/T_\infty)$ and proportionality factor $A(\theta, T_r/T_\infty)$ on the direction of radiation. Again, the data reveal substantial and obvious differences on their dependence on θ in the two sectors mentioned previously.

A survey of all available single-microphone far-field jet noise data inevitably indicate, individually and collectively, that there are two

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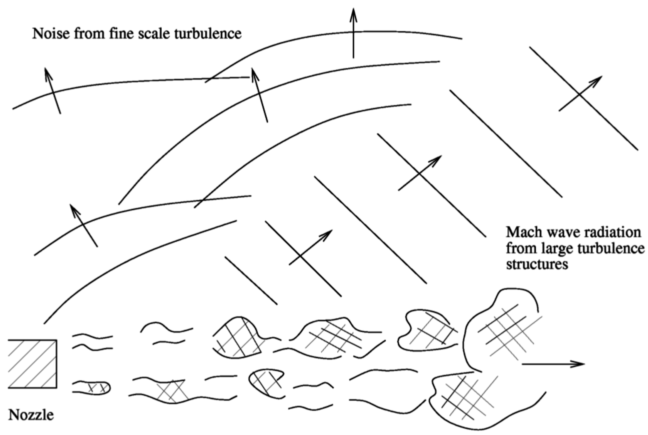


Fig. 1 The two noise fields and sources associated with a high-speed jet.

distinctly different sound fields associated with high-speed jets. The clear message is that there are two components of jet noise and two different noise sources.

Results of two-microphone far-field jet noise correlations are reported by Tam et al. [4]. The motivation of their measurement is that if, indeed, there are two distinct sound fields, they should be detectable in correlation measurements. In their experiment, an array of far-field microphones spaced at 10 deg apart was used. Measured results reveal significant normalized correlation when both microphones are in the downstream sector in which the large turbulence structure's noise dominates. When the microphones are outside this sector, especially at 90 deg, the normalized two microphone correlations drop to a very low level. This observation is in agreement with the earlier measurements of Maestrello [12,13] and the recent data of Viswanathan [14]. The observation is a confirmation that fine-scale turbulence noise is random and uncorrelated and that of the large turbulence structures is quite coherent and correlated spatially and temporally. These characteristics are simply a reflection of those of the noise sources.

The most direct experimental support for the two-source model of jet mixing noise is the direct correlation measurements of Panda and Seasholtz [15] and Panda et al. [16]. Some of the principal correlation data are also reported in [4], which offers a very detailed explanation of the measured data. Panda et al. [16] used a laser probe to measure the density and velocity fluctuations associated with the turbulence inside a high-speed jet by means of the Rayleigh scattering technique. They correlated the measured fluctuations with far-field microphone output. This provides a direct measure of the cause and effect of jet noise generation. Their data show large normalized correlations when the far-field microphone is in the downstream sector. But when the far-field microphone is moved to a sideline position, the correlation drops to the instrumentation noise level. The two-noise-source model of Fig. 1 suggests that the dominant source of noise radiating to the sideline and upstream directions is from the fine-scale turbulence of the jet flow. When measuring the turbulence

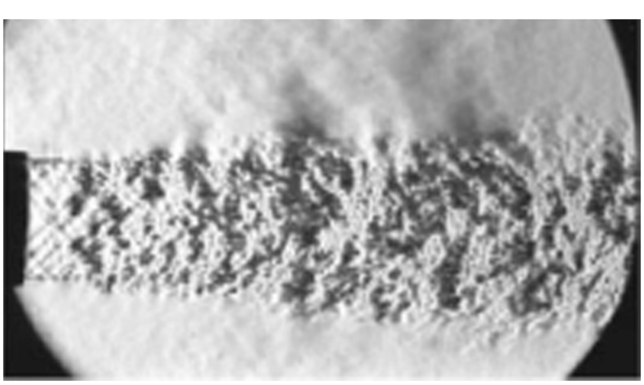


Fig. 2 Spark schlieren picture of a Mach 1.4 jet showing blobs of fine-scale turbulence.

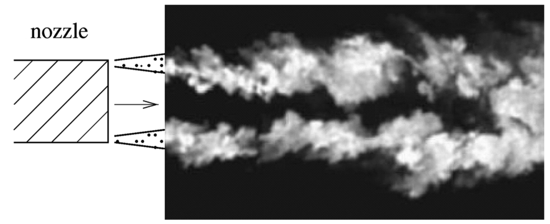


Fig. 3 Pulsed-laser picture of a Mach 1.3 jet [5] showing the presence of large turbulence structures.

fluctuations in a jet by the Rayleigh scattering technique, the fluctuations measured are associated with a small blob of turbulence. The pressure signal (measured by a far-field microphone at, say, 90 deg), however, comes from the numerous blobs of turbulence of the entire jet. Thus, the intensity of the acoustic pressure from the blob of turbulence in the measurement volume of the laser probe is miniscule. It is totally overwhelmed by the noise from all the other blobs of small-scale turbulence in the jet. Hence, one should expect very low or no correlation between far-field noise pressure and the laser probe signal inside the jet. The two-noise-source model also suggests that the large turbulence structure's noise is coherent and Mach-wave-like. It is highly directional and radiates principally in the downstream direction. Now the signal from the laser probe although very localized, is in fact representative of that of the large turbulence structures. The large turbulence structures radiate a significant fraction of the sound measured by the far-field microphone (say, at 150 deg). Therefore, the turbulent motion that generated the signal at the laser probe is also responsible for a significant fraction of the noise received by the microphone. Hence, according to this model, there should be reasonable correlation between the laser probe and the microphone signals. This explains why [4,16] found significant normalized correlation when the microphone was in the downstream sector and practically no correlation when it was put in the sideline direction.

The two-noise-source model, as originally conceived, envisages that large turbulence structures generate noise through Mach wave radiation. The objective of this paper is to review the Mach wave radiation processes and to provide theoretical and experimental support for the idea. In the far-field, Mach wave radiation cannot easily be distinguished from other forms of noise radiation. Therefore, most of the present paper concentrates on the acoustic near field of high-speed jets. It will be shown that in the near field, Mach wave radiation has a signature pressure-contour pattern. This pattern is established theoretically and verified experimentally. This signature pattern may serve as a detector for Mach wave radiation from a jet. The rest of this paper is as follows. Section II discusses the noise generation mechanism and source characteristics of large turbulence structures in high-speed jet flows.

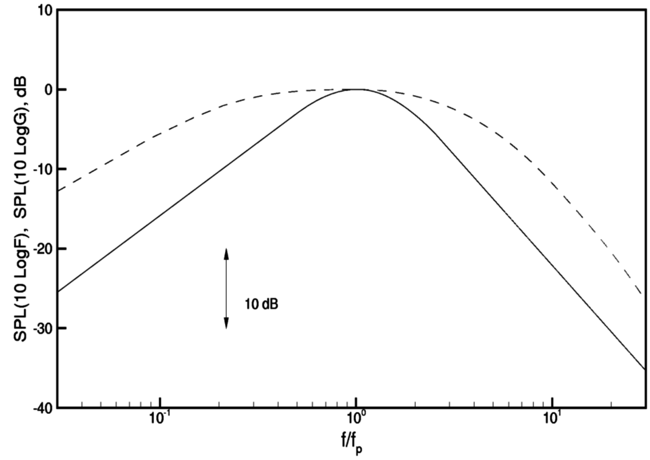


Fig. 4 The two similarity spectra of Tam et al. [3] (SPL denotes sound pressure level).

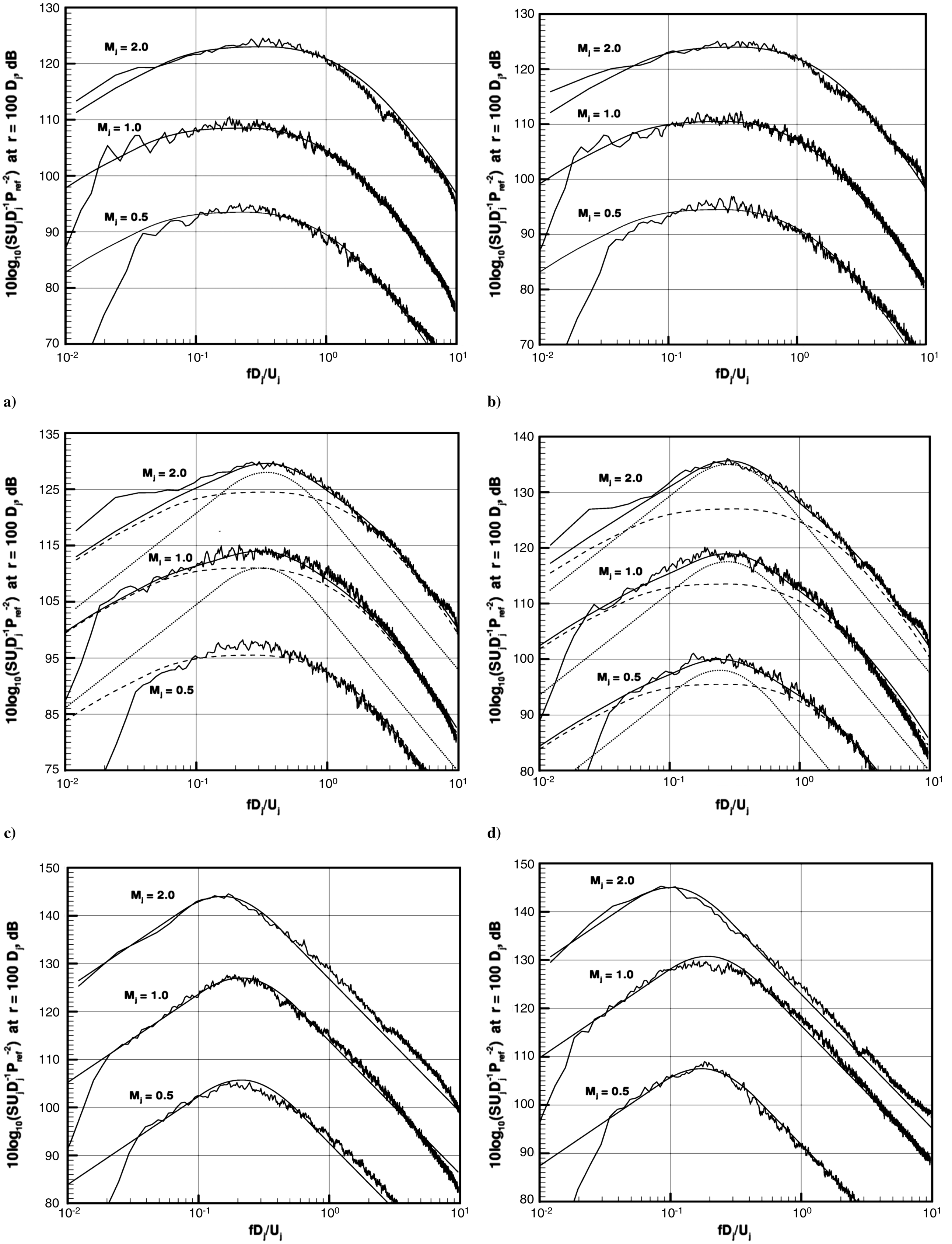


Fig. 5 Comparisons between similarity spectra and measured spectra by Seiner et al. [9] and Viswanathan [10]; $T_r/T_a = 3.2$: a) 90 deg, b) 100 deg, c) 110 deg, d) 120 deg, e) 130 deg, and f) 140 deg.

Section III is devoted to Mach wave radiation in the far field. Mach wave radiation is, however, more readily observable in the near field. This is discussed and analyzed in Sec. IV. Experimental evidence is provided to support the theoretically predicted Mach wave radiation pattern and the associated noise-source distribution. Section V summarizes and concludes the present study.

II. Noise Generation Mechanism and Source Characteristics of Large Turbulence Structures

What is the mechanism by which large turbulence structures generate noise? There have been a number of suggestions in the past. Mach wave radiation is one of the proposed mechanisms. In this paper, only this mechanism is investigated. Figure 6 is a shadowgraph of a Mach 2 cold jet showing intense Mach wave radiation. The idea of Mach wave radiation was discussed in considerable depth by Tam and Burton [17,18]. It was further developed and elaborated by Tam et al. [4]. The source of Mach wave radiation is turbulence that is chaotic and random. A stochastic description is therefore appropriate (see Tam and Chen [19]). As a simple model of large turbulence structures, one may regard them, in a stochastic sense, as traveling instability waves. When the wave speed is supersonic relative to the ambient speed of sound, the near pressure field of the large turbulence structures, which extends outside the jet, develops into Mach waves very similar to that of supersonic flow past a wavy wall. The source of Mach waves extends over a distance of many jet diameters downstream along the length of the jet, starting from the nozzle exit.

Tam and Burton [17,18] pointed out that in a jet, instability waves (large turbulence structures) begin with very small amplitude at the nozzle exit. They grow as they propagate downstream until they become damped. Further downstream, the waves decay. This growth and decay process or amplitude modulation is important, especially for Mach wave radiation from high-subsonic jets. To illustrate this point, let us represent an instability wave/large turbulence structure by a simple traveling wave; that is, the pressure associated with the wave may be represented by

$$p = Ae^{i(kx-\omega t)} \tag{1}$$

where ω is the angular frequency, k is the wave number, and A is the amplitude (only the real part is of concern to us). A constant-amplitude wave, such as that given by Eq. (1), behaves like a wavy wall. If the phase velocity ω/k is supersonic relative to ambient sound speed a_0, there will be Mach wave radiation. However, if the phase velocity is subsonic, there is no sound radiation. If the wave behaves like an instability wave with its amplitude undergoing growth and decay, then the wave effectively is not one that has a single wave number k. Instead, the wave has a broad band of wave numbers. To make this clear, suppose that $A(x)$ has the form of a Gaussian function with a half-width b centered at x_0; that is,

$$A(x) = A_0 e^{-(\ell n\, 2)(\frac{x-x_0}{b})^2} \tag{2}$$

By replacing A with $A(x)$ given by Eq. (2), it is easy to find

$$p(x,t) = A_0 e^{-(\ell n\, 2)(\frac{x-x_0}{b})^2 + i(kx-\omega t)} = \frac{A_0 b}{(4\pi\, \ell n\, 2)^{1/2}} \int_{-\infty}^{\infty} e^{-\frac{(\alpha-k)^2 b^2}{4\, \ell n\, 2} + i(\alpha x - \omega t) + i(k-\alpha)x_0} \mathrm{d}\alpha \tag{3}$$

Equation (3) shows that the wave no longer has one wave number k. It is now broadband, as shown in Fig. 7. The spectrum has a Gaussian shape centered at $\alpha = k$. Suppose that the original constant-amplitude wave is subsonic (i.e., $\omega/k < a_\infty$). In this case, the constant-amplitude wave will radiate no sound. But, with amplitude variation, a part of the spectrum could have supersonic phase velocity with $\omega/k > a_\infty$. This part of the spectrum (shaded in Fig. 7) will radiate sound. This is a mechanism by which large turbulence structures (instability waves) of high-subsonic jets radiate sound to the far field, as first suggested by Tam and Burton [18]. In a more recent work, Kopiev et al. [20] succeeded in measuring instability waves in a supersonic jet and its acoustic radiation. Their experiment offers a direct validation of the preceding instability-wave noise generation mechanism.

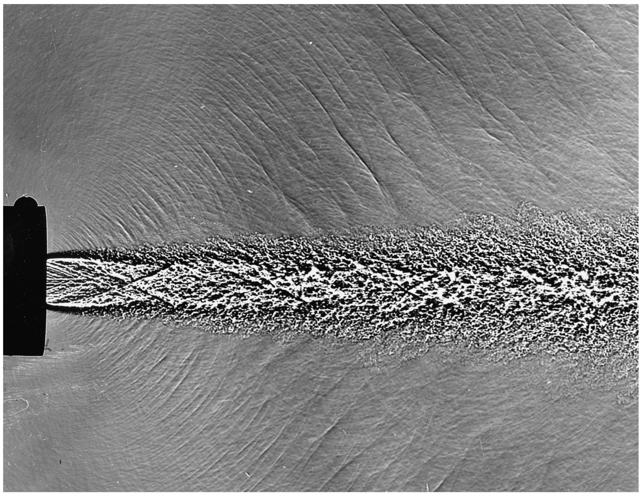
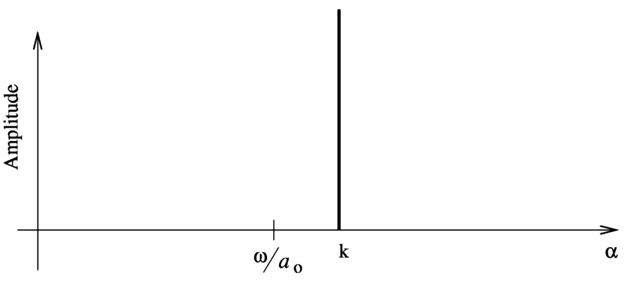


Fig. 6 Shadowgraph of a Mach 2.0 cold jet showing intense Mach wave radiation.

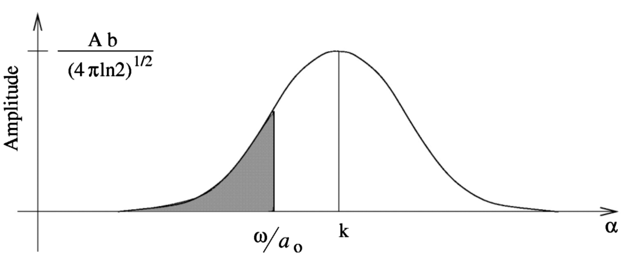
The preceding analysis indicates clearly that amplitude modulation is a crucial part of Mach wave radiation from large turbulence structures of the jet flow. This mechanism (see Fig. 7b) prevents an abrupt change in the noise radiated from a jet when the jet Mach number increases from subsonic through transonic range to supersonic. This would have been the case for constant-amplitude large turbulence structures (instability waves). Experimentally, the variation of jet noise with Mach number has been found to be smooth and continuous. Thus, it is important to recognize that amplitude modulation is a critical element of the Mach wave radiation mechanism.

III. Mach Wave Radiation Observed in the Far Field

Figure 8 shows Mach wave radiation generated by a wavy wall moving at a supersonic speed. If U_c is the speed of the wavy wall and a_∞ is the speed of sound, then the Mach wave radiation angle (inlet angle) is expected to be



a)



b)

Fig. 7 Wave number spectra: a) line spectrum of a constant-amplitude wave with a single wave number and b) broadband spectrum arising from wave amplitude variation. Shaded region contains waves with supersonic phase velocity.

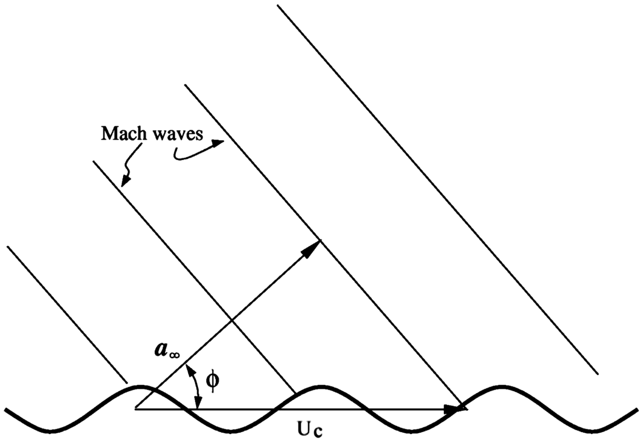


Fig. 8 Mach wave radiation from a supersonically traveling wavy wall.

$$\phi_{\text{peak}} = \pi - \cos^{-1}\left(\frac{a_\infty}{U_c}\right) \tag{4}$$

If, indeed, Mach wave radiation is the mechanism by which large turbulence structures of a high-speed jet generate noise, then one would expect the direction of the most intense noise radiation to be given by ϕ_{peak} of Eq. (4). To compute the Mach wave angle, one has to find the wave speed. Tam [21] computed the dispersion relation of the instability waves of a Mach 2.1 jet using a vortex-sheet jet model. The relationship is shown in Fig. 9. Also lotted in this figure are the measurements of Troutt and McLaughlin [22]. By means of the dispersion relation, the wave speed can be computed. The results are shown in Fig. 10. The agreement between theory and experiment is very good. Figure 10 indicates that over a broad range of Strouhal number, the wave speed for very-high-speed jets is close to 80% of the jet exit velocity U. By taking $U_c = 0.8U$, the Mach wave angle of Eq. (4) may now readily be computed.

Figure 11 shows a set of jet noise directivity measured recently by Petitjean et al. [23] for a Mach 1.92 jet at temperature ratio T_j/T_a of 0.58, 1.32, and 1.65. The directivity of each of the jets exhibits a well-defined peak. The Mach angles of these jets computed according to Eq. (4) are 149, 124.6, and 120.4 deg, respectively. These angles are indicated by arrows in Fig. 11. It is obvious that these values are in good agreement with the measurements. The good agreement lends support to the belief that Mach wave radiation from the large turbulence structures is a dominant jet-noise-generation mechanism.

IV. Mach Wave Radiation in the Near Field

It is useful to point out that the characteristic Mach wave radiation pattern of Fig. 6 can only be seen in the jet near field. When the acoustic waves propagate to the far field they become outgoing waves with the same characteristics as other outgoing sound waves. The wave front spreads out and the amplitude decreases inversely as the distance from the jet exit increases. Of interest to us here are Mach waves in the near field.

In a study of Mach wave radiation pattern in the near field associated with an instability wave of a supersonic jet, Tam [24] obtained an analytical solution using a vortex-sheet jet model. In this model, (see Fig. 12), the jet is bounded by a vortex sheet. It is excited by a localized time periodic force at the nozzle exit. The mathematical problem is as stated next:

$$r > R, \qquad \frac{\partial^2 p_+}{\partial t^2} = a_0^2 \nabla^2 p_+ \tag{5}$$

$$r < R, \qquad \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)^2 p_- = a_j^2 \nabla^2 p_- \tag{6}$$

At $r = R$,

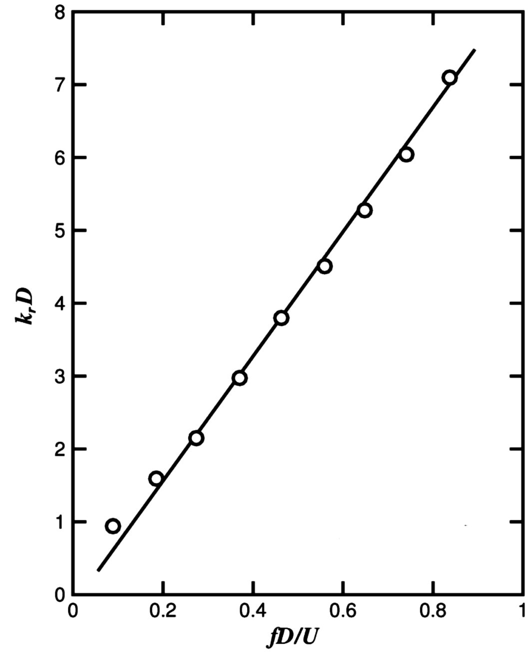


Fig. 9 Dispersion relation of the instability waves of a Mach 2.1 cold jet according to the vortex-sheet jet model; theory [21] (solid line) and experiment of Troutt and McLaughlin [22] (circles).

$$-\frac{1}{\rho_+}\frac{\partial p_+}{\partial r} = \frac{\partial^2 \varsigma}{\partial t^2} \tag{7}$$

$$-\frac{1}{\rho_-}\frac{\partial p_-}{\partial r} = \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)^2 \zeta \tag{8}$$

$$p_+ = p_- + A\exp[i(n\theta - \omega t)]\delta(x) \tag{9}$$

where (p_+, ρ_+, and a_o) and (p_-, ρ_-, and a_j) are the pressure, density, and sound speed outside and inside the jet, and ζ is the displacement of the vortex sheet. The preceding problem can be solved analytically by applying the Fourier–Laplace transform to the x and t variables. It is found (see [24]) that the Kelvin–Helmholtz instability wave is given by the root of the following dispersion relation:

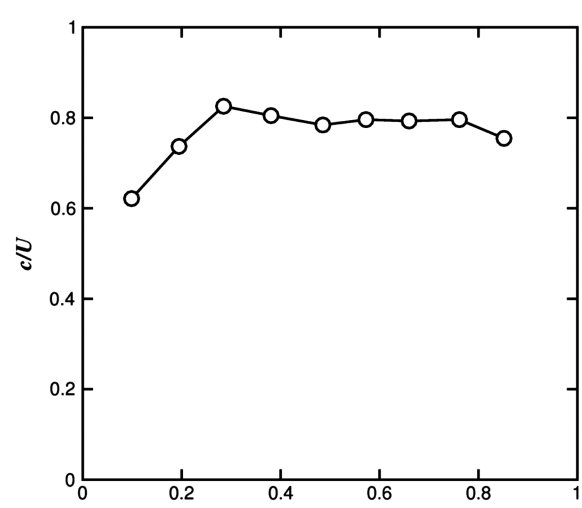


Fig. 10 Wave speed, $c = \omega/k$, of the instability waves of a Mach 2.1 cold vortex-sheet jet; Troutt and McLaughlin [22] (circles).

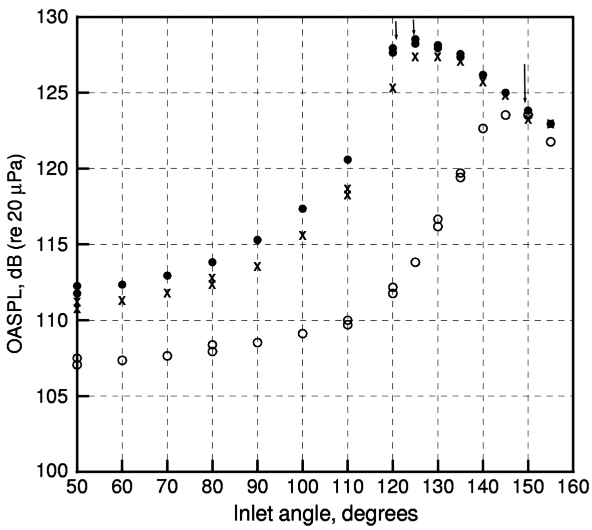


Fig. 11 OASPL directivity patterns of a Mach 1.92 jet at $T_j/T_a = 0.58$ (open circles), 1.32 (crosses), and 1.65 (filled circles); data from Petitjean et al. [23].

$$F(c) \equiv c^4 - 2\left(\frac{u}{a_0}\right)c^3 + \left[\left(\frac{u}{a_0}\right)^2 - \left(\frac{a_j}{a_0}\right)^2 - 1\right]c^2 + 2\left(\frac{u}{a_0}\right)c - \left(\frac{u}{a_0}\right)^2 = 0 \qquad (10)$$

where $c = \omega/ka_0$, and u is the jet velocity. Equation (10) has four roots for c: two real and two complex conjugate. Kelvin–Helmholtz instability is given by the complex root with positive imaginary part ($c = c_r + ic_i$ for $c_i > 0$). The pressure pattern outside the jet with respect to a cylindrical coordinate system (r, θ, x) with x coinciding with the jet axis is

$$p(r,\theta,x,t) \sim \text{constant} \times \exp\left\{\frac{c_i x - (c_r\beta_i - c_i\beta_r)(r - (D/2))}{c_r^2 + c_i^2}\frac{\omega}{a_0}\right\} \times \exp\left\{i\left[\frac{c_r x + (c_r\beta_r + c_i\beta_i)(r-(D/2))}{c_r^2 + c_i^2}\right]\frac{\omega}{a_0} + in\theta - i\omega t\right\} \quad x > 0 \qquad (11)$$

where D is the diameter of the jet, n is the azimuthal mode number, $c = c_r + ic_i$ is the unstable root of Eq. (10), and $\beta_r + i\beta_i = (c^2 - 1)^{1/2}$ with $\beta_i > 0$.

Equation (11) is a spatially amplifying wave solution. The solution consists of a spatially dependent amplitude [the first exponential function of Eq. (11)] and a wavelike oscillatory part [the second exponential function of Eq. (11)]. From the amplitude function, it is straightforward to find that Mach waves are exponentially small to the left of the line:

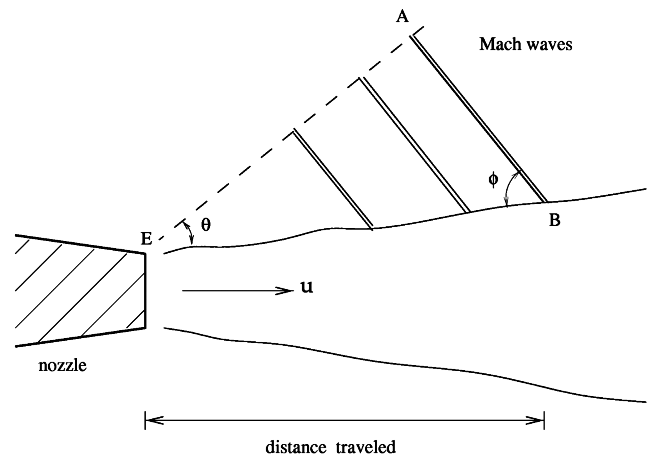


Fig. 13 Schematic diagram of Mach wave radiation pattern associated with the instability waves of a supersonic jet.

$$c_i x - (c_r\beta_i - c_i\beta_r)\left(r - \frac{D}{2}\right) = 0 \qquad (12)$$

This is line AE in Fig. 13 (see also the wave pattern in Fig. 14). On a plane $\theta =$ constant, the Mach waves are straight lines parallel to line AB in Fig. 13. They propagate in the direction normal to the wave front. The equation for the parallel lines is obtained by setting the phase function of Eq. (11) to a constant; that is,

$$c_r x + (c_r\beta_r + c_i\beta_i)\left(r - \frac{D}{2}\right) - (c_r^2 + c_i^2)a_0 t = \text{constant} \qquad (13)$$

This pattern is very similar to the optical picture in Figs. 6 and 14.

The angle ϕ of the Mach waves is given by

$$\tan\phi = \frac{c_r\beta_r + c_i\beta_i}{c_r} \qquad (14)$$

Figure 15 shows a comparison of the angle ϕ computed according to Eq. (14) and the experimental measurements of Rosales [25] and Lowson and Ollerhead [26] for supersonic nitrogen jets. The angle ϕ is around 50 deg for these jets in the Mach number range measured. Figure 16 is a comparison between the computed angle ϕ and the measured angle by Rosales [25] for helium jets. The angle ϕ for helium jets is around 30 to 35 deg. There is good agreement between predictions and measurements.

The wave fronts of Mach waves are fairly coherent. The source of each line disturbance is embedded in the jet flow [e.g., for AB, it is at B (see Fig. 13)]. To generate wave front AB, it is necessary for the source at B to start producing the Mach line AB when it exits the nozzle at E. So that the wave front remains continuous, it is further necessary for the source to keep generating a Mach wave as it travels from nozzle exit E to B. The distance of travel, as shown in Fig. 13, is a number of jet diameters long. This consideration suggests that the source of Mach wave radiation must have a long lifetime, must be very energetic and fairly coherent, and must extend over many jet

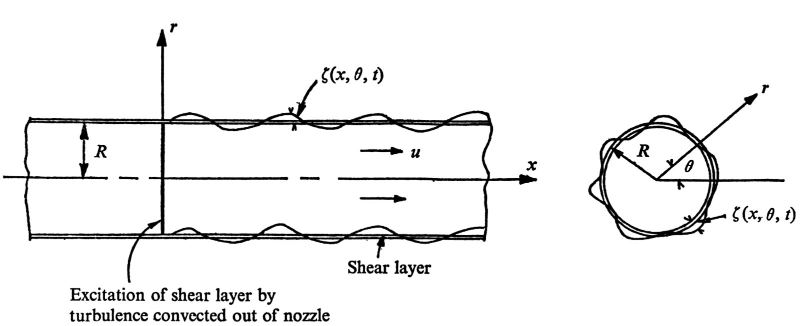


Fig. 12 Vortex-sheet model of a high-speed jet excited by a localized periodic force.

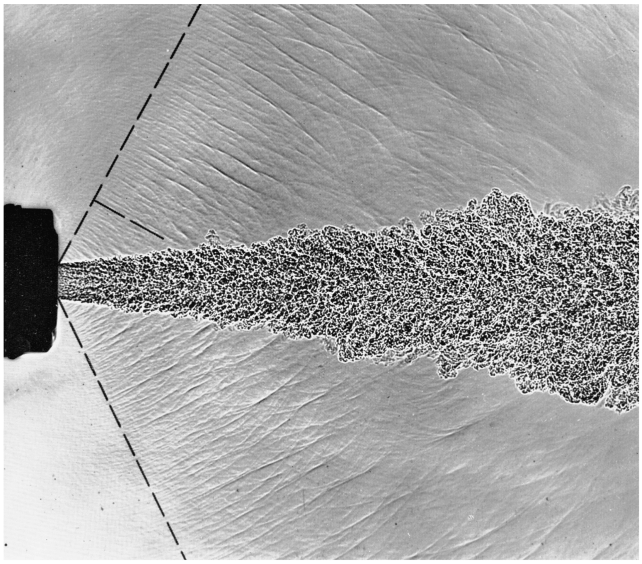


Fig. 14 Mach wave radiation pattern of a Mach 1.5 helium jet.

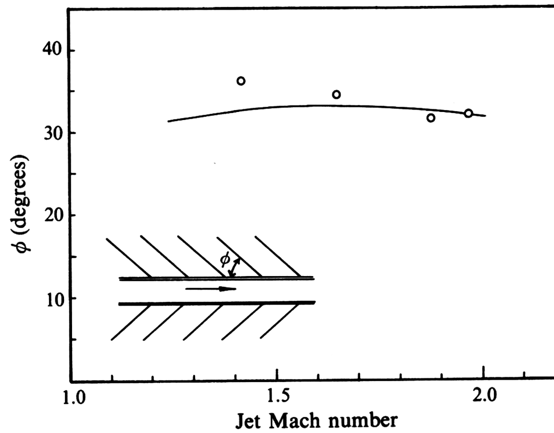


Fig. 16 Comparison between computed [Eq. (14)] and measured wavefront angle of the Mach wave radiation pattern outside supersonic helium jets; Eq. (14) (solid line) and experiment of Rosales [25] (circles).

diameters downstream. Small blobs of turbulence (fine-scale turbulence) will definitely not qualify. The most likely sources are instability waves (large turbulence structures).

Because the large turbulence structure's noise radiates principally in the downstream direction, let us now examine the distributions of noise sources radiating to large inlet angles. Recently, Viswanathan (see [4]) measured the noise-source distributions of high-speed jets radiating to 150 deg inlet angles. The measurements were made using a mirror microphone. The results of four jets are shown in Fig. 17. If we take the length of the noise source to be defined by noise level within 2 dB of the maximum, then the length of the source region is long, compared with the potential core length of the jet. For the Mach 2 jet, the source region has a length of 14 jet diameters. Using the same criterion, the noise-source region radiating to 90 deg has a length of 9 jet diameters. Thus, the source of Mach wave radiation is nearly one-and-a-half times as long as the noise source of the fine-scale turbulence. For the lower-Mach-number jets, the length of the source region is over 6 jet diameters. The source is not localized. This fact is consistent with Mach wave radiation, which is the dominant noise generation mechanism for noise radiating to large inlet angle directions. Note that the data of Fig. 17 show only consistency, not a proof of Mach wave radiation.

One important shortcoming of vortex-sheet jet models is that the instability-wave amplitude will grow indefinitely spatially. In a realistic jet, because of the spreading of the jet mixing layer, an instability wave of a specific frequency will eventually become damped once it reaches the part of the jet in which the mixing layer is too thick to support such instability. Thus, all instability waves will undergo a growth and decay process. The wave decay process affects the near-field Mach wave pattern substantially. Tam and Burton [18] obtained a complete instability-wave growth and decay solution using the method of matched asymptotic expansions. Figure 18a, taken from their work, shows a typical near-field pressure-contour pattern associated with an instability wave of a high-speed jet. The pattern is characterized by parallel, closely spaced contours on the lower left side of the contour pattern starting near the nozzle exit. The contours are closely spaced and nearly parallel because near line AE of Fig. 13 the wave amplitude decays exponentially fast. Contour lines are practically parallel and closely spaced at 90 deg to a steep gradient. Another salient feature of Fig. 18a is the contour loops,

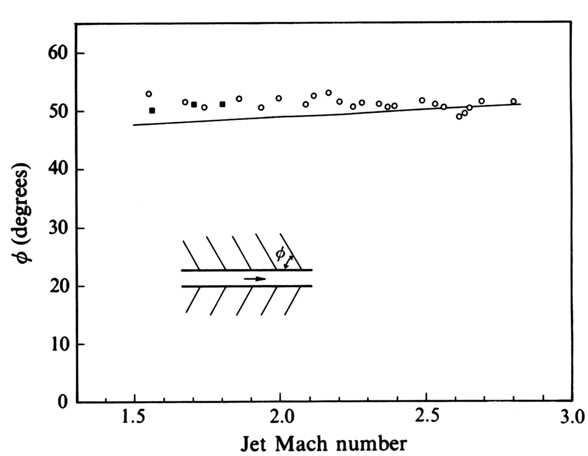


Fig. 15 Comparison between computed [Eq. (14)] and measured wavefront angle of the Mach wave radiation pattern outside supersonic nitrogen jets; Eq. (14) (solid line) and experiments of Rosales [25] (circles) and Lowson and Ollerhead [26] (squares).

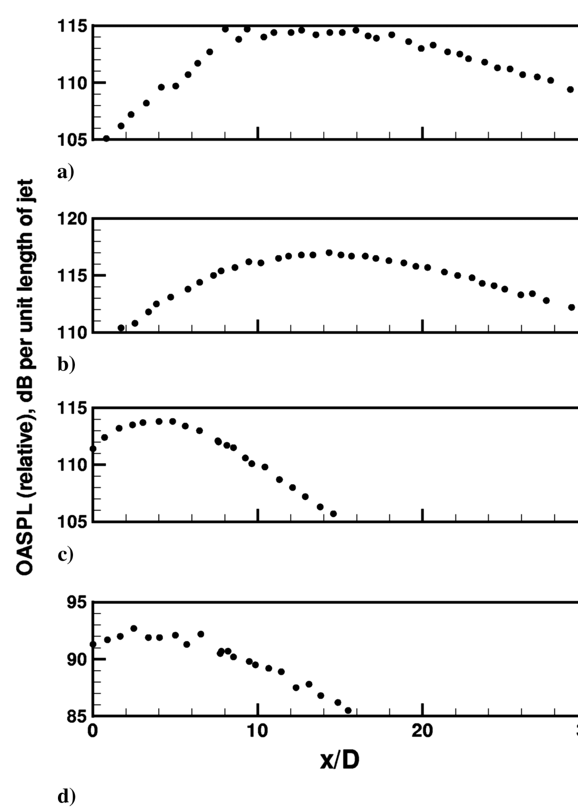


Fig. 17 Distribution of noise sources radiating to the 150 deg direction for four jets. Data measured by Viswanathan [4] using a mirror microphone: a) $M = 1.9$ and $T_r/T_a = 1.0$, b) $M = 1.9$ and $T_r/T_a = 2.2$, c) $M = 0.9$ and $T_r/T_a = 3.2$, and d) $M = 0.5$ and $T_r/T_a = 3.2$.

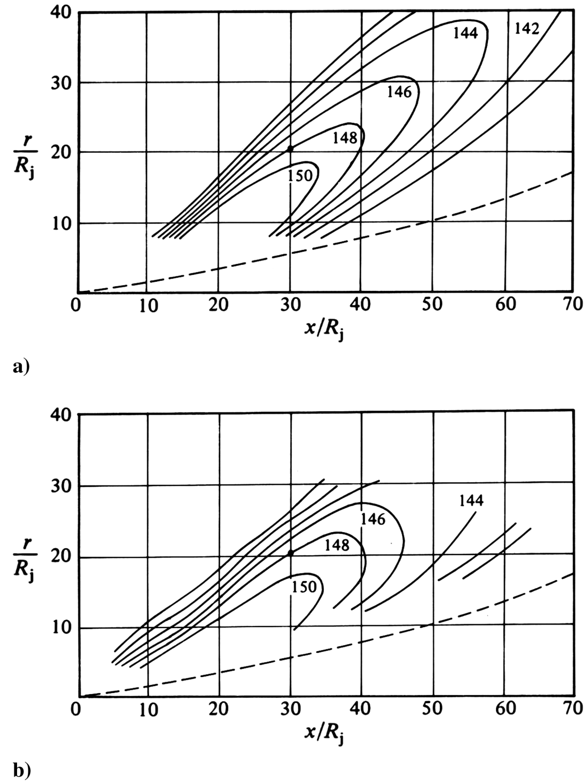


Fig. 18 Near-field pressure-contour pattern associated with Mach wave radiation. Mach 2.1 cold jet at Strouhal number 0.4: a) computed results of Tam and Burton [18] and b) experimental measurements of Troutt and McLaughlin [22].

which are due to the saturation and decay of the instability wave. The loops point in the direction of maximum noise radiation. We believe that the pressure contours of Fig. 18a form a telltale characteristic near-field pattern of Mach wave radiation. In other words, this is the signature pattern of Mach wave radiation. The characteristic near-field pattern is in good agreement with the measurements of Troutt and McLaughlin [22] shown in Fig. 18b.

The near-field pressure-contour pattern contains a good deal of information about the noise source of a jet. In particular, it could provide evidence in support of Mach wave radiation even for jets with subsonic convective Mach numbers. The near-field contour data may also be used indirectly to estimate the location and extent of the noise-source region. Extensive near-field fluctuating pressure-contour maps for a Mach 1.5 jet with a nozzle exit diameter of 0.408 in. was measured by Yu [27] some time ago. For this jet, the

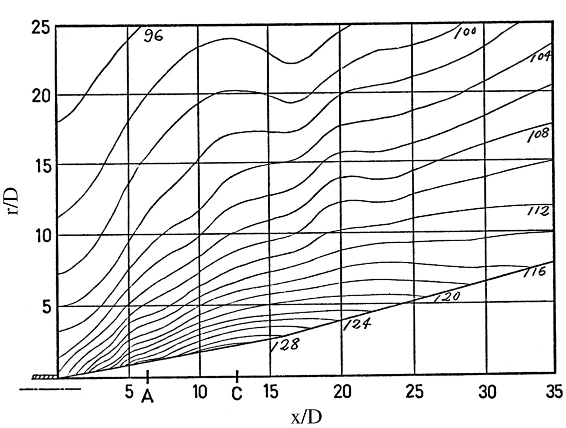


Fig. 19 Near-field pressure contours of a Mach 1.5 cold jet measured by Yu [27] at one-third-octave-band center frequency of 2 kHz. *A* is the location of the end of the potential core. *C* is the location of the end of the supersonic core.

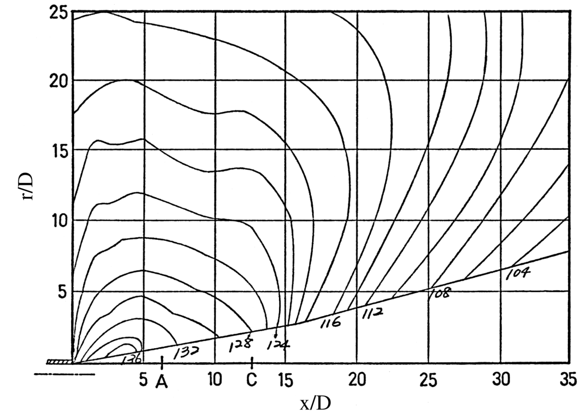


Fig. 20 Near-field pressure contours of a Mach 1.5 cold jet measured by Yu [27] at one-third-octave-band center frequency of 100 kHz.

potential core extends to 6.5 jet diameters downstream. The supersonic core ends at 12.5 diameters downstream. Figure 19 is a one-third-octave-band pressure-contour map at 2.0 kHz center frequency. In this figure, the jet centerline is at 0.5 units (1 jet diameter is a unit) below the $r/D = 0$ lines. The outer edge of the jet flow, starting from the nozzle lip, is shown as the boundary curve to which many pressure contours end. From the distribution of the pressure contours, it is clear that the noise source at 2 kHz peaks around 12 diameters downstream, near the end of the supersonic core. This is typical of low-frequency noise sources; they lie quite far downstream. The source is fairly localized. Figure 20 is a one-third-octave-band pressure-contour map at 100 kHz center frequency. The source region is again localized. It is centered very close to the nozzle exit, around 4 diameters downstream. This is typical of high-frequency noise sources; they are located quite close to the nozzle exit. The frequency at the peak of the noise spectrum of the Mach 1.5 jet is around 10 kHz. Figure 21 is the one-third-octave-band pressure-contour map at this frequency. The contour pattern is distinctively different from those at low and high frequencies. Judging from the distribution of pressure contours very close to the edge of the jet (the 138 dB contour), one easily draws the conclusion that the source region is not localized. It extends approximately from $x = 4.5D$ to $16D$ downstream. This set of data suggests that the noise source responsible for peak noise radiation is distributed over a considerable length of the jet. This is consistent with Mach wave radiation. Note that for the $M = 1.5$ number jet of Fig. 21, if the convection speed of the large turbulence structures is taken to be 0.7 of the jet exit velocity (a good empirical estimate for cold jets at moderate supersonic Mach numbers), then the convective Mach number u_c/a_0 is high subsonic.

By comparing Figs. 18a and 21, it is easy to note the striking resemblance. That is to say, the near-field pressure-contour pattern of a jet at a high-subsonic convective Mach number is very similar to the signature Mach wave radiation pattern. Both patterns exhibit close

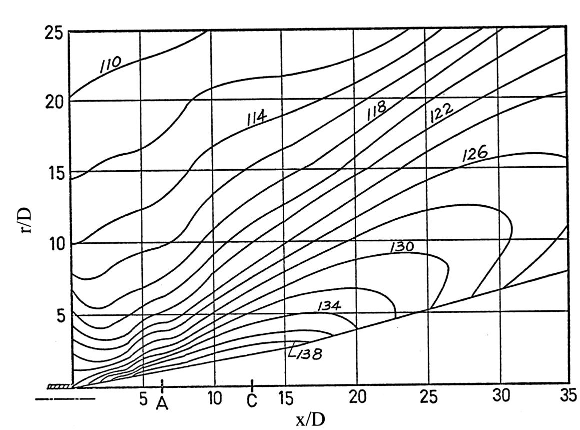


Fig. 21 Near-field pressure contours of a Mach 1.5 cold jet measured by Yu [27] at one-third-octave-band center frequency of 10 kHz.

parallel contours, starting from the nozzle lip, and a distinct loop. These features are absent in Figs. 19 and 20. We believe that although the convective Mach number of a cold Mach 1.5 jet is high subsonic, the dominant part of its noise is generated by Mach wave radiation as a result of amplitude modulation.

V. Conclusions

Mach wave radiation is investigated as a possible mechanism by which large turbulence structures of high-speed jets radiate intense noise. Mach wave radiation by the large turbulence structures of high-speed jets is similar to sound generated by a supersonically traveling wavy wall. However, the growth and decay of the large turbulence structures must be taken into consideration. It is shown that this amplitude-modulation mechanism does allow subsonically traveling large turbulence structures to generate Mach waves through a broadening of the wave number spectrum.

The far and near-field Mach wave radiation characteristics are examined. Experimental results on the peak direction of noise radiation for high-supersonic jets are considered. They are shown to agree well with the computed Mach wave radiation direction. A near-field characteristic signature pressure-contour pattern for Mach wave radiation is identified. The theoretical signature pattern is shown to agree well with near-field experimental measurements for both a high-supersonic jet and for a jet at high-subsonic convective Mach numbers. It is suggested that this characteristic signature pattern may be used as an indicator for Mach wave radiation.

It is established in this investigation that the source responsible for Mach wave radiation cannot be in a localized region of a jet. The source must be distributed over a considerable length of the jet column relative to the potential core length. Experimental support for this conclusion is provided both by direct noise-source measurements using a mirror microphone and by near-field pressure-contour pattern.

In the literature, a number of proposals have been made concerning how noise is generated by the large turbulence structures of high-speed jets. In this work, a comprehensive set of theoretical and experimental results is presented in favor of Mach wave radiation. Further work is, without a doubt, desirable. However, the scope of the present study is quite large and the agreements with experimental measurements are consistently good. This offers confidence in the belief that Mach wave radiation is at least one of the dominant mechanisms, if not the most dominant mechanism, responsible for noise radiation from the large turbulence structures of high-speed jets.

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